

# Modelling of Power System Components

## LEARNING OBJECTIVES

After a careful study of the chapter, you would be able to understand:

- Modelling of transmission lines.
- Modelling of in-phase transformers.
- Modelling of phase-shifting transformers.
- Modelling of excitation systems.
- IEEE models for excitation systems.
- Modelling of turbines.
- Static and dynamic load models.
- Modelling of synchronous machines.

## 2.1 Modelling of Transmission Lines

The modelling of power system components plays an important role in all EMS functions. A proper choice of a model is important and determines the accuracy, computation time, memory requirements, etc., in any application. In all EMS tools such as load flow analysis, stability analysis, state estimation, contingency analysis, load frequency control, etc., the  $\pi$  model of the transmission line is used. The  $\pi$  model of the transmission line is shown in Fig. 2.1. This chapter briefly discusses the modelling of various power system components.

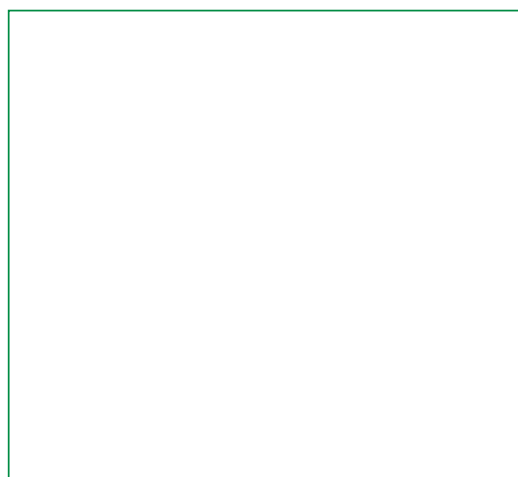


Figure 2.1  $\pi$  model of a transmission line.

The branch series impedance is given by

$$z_{ij} = r_{ij} + jx_{ij} \quad (2.1)$$

$$y_{ij} = z_{ij}^{-1} = g_{ij} + jb_{ij} \quad (2.2)$$

where

$$g_{ij} = \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} \quad \text{and} \quad b_{ij} = -\frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} \quad (2.3)$$

Apart from the series impedance, we also have the shunt admittance that is expressed as

$$y_{sh} = g_{sh} + jb_{sh} \quad (2.4)$$

In actual transmission lines, the series impedance components  $r_{ij}$  and  $x_{ij}$  have positive values. This means that  $g_{ij}$  is positive and  $b_{ij}$  is negative. In real line sections,  $g_{sh}$  and  $b_{sh}$  are both positive. However, in reduced models, used extensively in stability studies and contingency analysis, the parameters may have a sign that is different from the normal line section. From Fig.2.1, the currents can be expressed as

$$I_{ij} = y_{ij}(V_i - V_j) + y_{sh}V_i \quad (2.5)$$

$$I_{ji} = y_{ij}(V_j - V_i) + y_{sh}V_j \quad (2.6)$$

where

$$V_i = |V_i| e^{j\delta_i}$$

$$V_j = |V_j| e^{j\delta_j}$$

### Example 2.1

The series impedance of a 138 kV transmission line is  $z = r + jx = 0.0068 + j0.0380$ . The total shunt susceptance is given by  $b_{sh} = 0.0108$  pu. The series impedance of a 750 kV line is  $z = 0.0007 + j0.0169$  pu and  $b_{sh} = 8.75$  pu. Find the  $x/r$  ratio and the  $b/b_{sh}$  ratio of both the lines. Comment on the results.

### Solution

Consider the 138 kV line  $z = 0.0068 + j0.0380$ .

$$x/r = 0.038/0.0068 = 5.59$$

$$b = \frac{-x}{r^2 + x^2} = \frac{-0.0380}{0.0068^2 + 0.038^2} = -25.5$$

$$\frac{b}{b_{sh}} = \frac{-25.5}{0.0108} = -2361.11$$

Consider the 750 kV line  $z = 0.0007 + j0.0169$ ,  $x/r = 0.0169/0.0007 = 24.14$ .

$$b = \frac{-x}{r^2 + x^2} = \frac{-0.1690}{0.0007^2 + 0.0169^2} = -59.07$$

$$\frac{b}{b_{sh}} = \frac{-59.07}{8.75} = -6.75$$

The 750 kV line has a much higher value of  $x/r$  than the 138 kV line. This implies a higher degree of decoupling between the active and reactive parts of the power flow problem. The magnitude of the  $b/b_{sh}$  ratio is smaller in the 750 kV line, implying that there is a need for some sort of compensation.

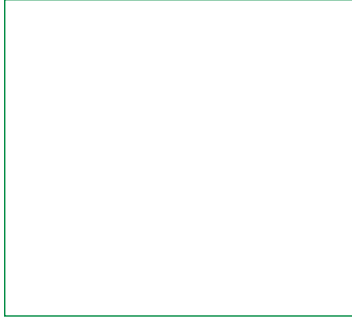


Figure 2.2 Transformer model.

## 2.2 Modelling of Transformers

Consider a transformer connected as shown in Fig. 2.2. The complex turns ratio is given by

$$t_{ij} = a_{ij} e^{j\delta_a} \quad (2.7)$$

The alternative model is shown in Fig. 2.3, where the tap is on the sending-end bus. In the figure,

$$\bar{t}_{ij} = a_{ij}^{-1} e^{-j\delta_b} \quad (2.8)$$



Figure 2.3 Transformer model with tap on sending-end side.

### 2.2.1 In-Phase Transformer

If we have an in-phase transformer (phase-shift is zero), the model is as shown in Fig. 2.4.

$$\left| \frac{V_k}{V_i} \right| = a_{ij}$$

Since we have an in-phase transformer there is no phase shift. Therefore,  $\delta_k = \delta_i$ . There is no loss (active and reactive) in the ideal transformer (the  $i - k$  part in Fig. 2.4). This means that

$$V_i I_{ij}^* + V_k I_{ji}^* = 0 \quad (2.9)$$

Further, since  $\delta_k = \delta_i$ , the complex voltage ratio is given by

$$\frac{V_k}{V_i} = \frac{|V_k| e^{j\delta_k}}{|V_i| e^{j\delta_i}} = a_{ij} \quad (2.10)$$

From Eqs. (2.9) and (2.10)

$$\frac{I_{ij}}{I_{ji}} = -\frac{|I_{ij}|}{|I_{ji}|} = -a_{ij} \quad (2.11)$$

The negative sign implies that the complex currents  $I_{ij}$  and  $I_{ji}$  are out of phase by  $180^\circ$ . The equivalent  $\pi$  model of the in-phase transformer is shown in Fig. 2.5.

We equate the quantities from Figs. 2.4 and 2.5 to obtain the equivalent circuit parameters. From Fig. 2.4, we have:

$$\frac{I_{ij}}{I_{ji}} = -a_{ij}$$

$$I_{ji} = (V_j - V_k) y_{ij} = y_{ij} V_j - a_{ij} V_i y_{ij} \quad (2.12)$$

$$I_{ij} = -a_{ij} I_{ji} = -a_{ij} y_{ij} V_j + a_{ij}^2 V_i y_{ij} \quad (2.13)$$

From Fig. 2.5

$$I_{ij} = (V_i - V_j) y_1 + V_i y_2 = V_i (y_1 + y_2) - V_j y_1 \quad (2.14)$$

$$I_{ji} = (V_j - V_i) y_1 + V_j y_3 = V_i (y_1 + y_3) - V_j y_1 \quad (2.15)$$

Comparing Eqs. (2.13) and (2.14), we have

$$y_1 + y_2 = a_{ij}^2 y_{ij}$$

$$y_1 = a_{ij} y_{ij}$$

Therefore,  $y_2 = a_{ij}^2 y_{ij} - a_{ij} y_{ij} = a_{ij} (a_{ij} - 1) y_{ij}$



Figure 2.4 In-phase transformer model.

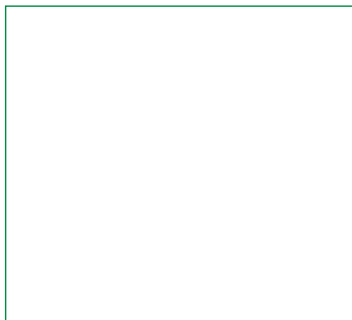


Figure 2.5  $\pi$  model of the in-phase transformer.



Figure 2.6 Elements of the  $\pi$  model of the in-phase transformer.

Comparing Eqs. (2.15) and (2.12), we get

$$y_1 + y_3 = y_{ij}$$

$$y_3 = y_{ij} - a_{ij}y_{ij} = y_{ij}(1 - a_{ij})$$

Thus, the equivalent  $\pi$  model of the in-phase transformer is shown in Fig. 2.6.

### 2.2.2 Phase-Shifting Transformer

The phase-shifting transformer is shown in Fig. 2.7. It is used to control active power flow, where the control variable is the phase angle. A phase-shifting transformer affects both the phases and magnitudes of  $V_i$  and  $V_k$ . Their ratio is not changed.

$$\frac{V_k}{V_i} = t_{ij} = a_{ij}e^{j\phi_{ij}} \quad (2.16)$$

Because of the phase shift,

$$\delta_k = \delta_i + \phi_{ij}$$

$$|V_k| = |V_i| a_{ij}$$

Using Eqs. (2.9) and (2.16), we get

$$\frac{I_{ij}}{I_{ji}} = -t_{ij}^* = -a_{ij}e^{-j\phi_{ij}} \quad (2.17)$$

The complex currents can be expressed in terms of complex voltages as

$$I_{ij} = -t_{ij}^*y_{ij}(V_j - V_k) = y_{ij}V_i - t_{ij}^*y_{ij}V_j \quad (2.18)$$

$$I_{ji} = y_{ij}(V_j - V_k) = -t_{ij}y_{ij}V_i + y_{ij}V_j \quad (2.19)$$

We cannot determine the components of the  $\pi$  model, since the coefficient of  $V_j$  that is  $-t_{ij}^*y_{ij}$  in Eq. (2.18) is different from the coefficient  $-t_{ij}y_{ij}$  of  $V_i$  in Eq. (2.19), as long as the phase shift is non-zero. So, the  $\pi$  model parameters cannot be obtained as the required symmetry is missing.

#### Concept Check

1. Why is the  $\pi$  model preferred for the transmission lines?
2. Derive the model for the transmission line.
3. The shunt admittance can be neglected. Comment.
4. Derive the  $\pi$  model of a transformer. What are its limitations?
5. How is a phase-shifting transformer modelled?

## 2.3 Modelling of Excitation Systems

The field winding of the AC generators needs a DC excitation. We need a device to supply the DC, which is done by the exciter. The voltage regulator controls the output of the exciter so that the required voltage is generated. The different types of exciters are:

1. DC exciters which use DC generators.
2. AC exciters which use AC generators in conjunction with rectifiers.
3. Static exciters which are exciters without any rotating elements.
4. Brushless exciters which have rotating rectifiers.

The general block diagram of an excitation is shown in Fig.2.8.

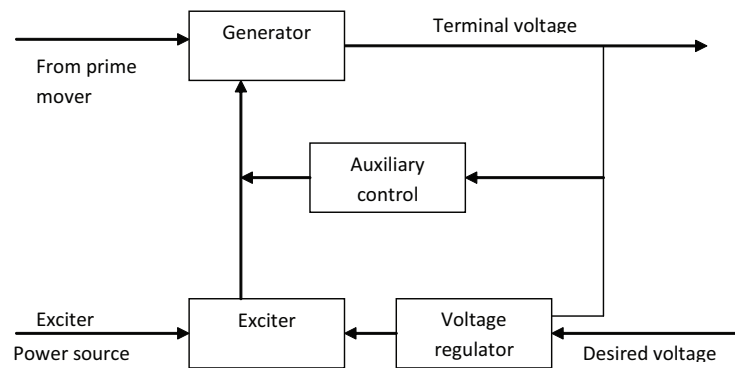


Figure 2.8 Excitation system.

The desired voltage or reference voltage is compared with the actual terminal voltage of the generator. The voltage regulator controls the exciter output such that the terminal voltage of the AC generator equals the desired voltage. Auxiliary controls are desirable for feedback of speed, frequency, acceleration and other signals to improve system conditions such as stability, damping, etc. Some standard definitions used in excitation systems are as follows[11]:

1. **Exciter:** The source of field current for the excitation of the generator.
2. **DC generator exciter:** An exciter whose energy is derived from a DC generator. The exciter itself may be driven by a motor, by another prime mover or by the shaft of the synchronous machine whose field is supplied by the exciter.
3. **Alternator-rectifier exciter:** An exciter whose energy is derived from an alternator and converted into DC by rectifiers. The alternator may be driven by a motor, prime mover or the shaft of the synchronous machine. The rectifiers may be stationary or rotating with the alternator shaft.
4. **Compound rectifier exciter:** An exciter whose energy is derived from the currents and potentials of the AC terminals of the synchronous machine and converted into DC by rectifiers. The exciter includes the power transformers, power reactors, power rectifiers including the gate circuitry.
5. **Potential source rectifier exciter:** An exciter whose energy is derived from a stationary AC potential source and converted into DC by rectifiers.
6. **Pilot exciter:** The source of field current for the excitation of another exciter.
7. **Continuously acting regulator:** A regulator that initiates corrective action for a sustained infinitesimal change in the controlled variable.
8. **Non-continuously acting regulator:** A regulator that initiates corrective action for a finite change in the controlled variable.
9. **Air gap line:** The extended linear part of the no-load saturation curve.
10. **Exciter ceiling voltage:** The maximum voltage that may be attained by an exciter under specified conditions. This puts a limit on the drop in terminal voltage that can be compensated by an increase in excitation.
11. **Excitation system voltage-time response:** The excitation system output voltage expressed as a function of time, under specified conditions.
12. **Excitation system voltage response time:** The time in seconds for the excitation voltage to reach 95% of the ceiling voltage.

The voltage regulator is normally a continuously acting system, which takes corrective action for any deviation in the AC terminal voltage. Since large-scale use of computers is prevalent in all control centres, the modelling of the excitation system gained importance. The IEEE Committee has suggested standard computer models for different types of excitation systems, which are adequate to represent all modern systems.

### 2.3.1 Exciters

The exciter is the device that feeds DC supply to the main generator field. In this section, discuss briefly some standard exciters have been briefly discussed.

#### DC Excitation Systems

These systems utilize a DC generator for the DC supply to the main generator field. Different configurations are possible as follows:

1. **Main exciter only:** The arrangement of an excitation system with a main exciter is shown in Fig.2.9(a). The exciter is a DC shunt generator whose output is connected to the alternator field through slip rings. This system is commonly used with small AC generators. The response of this system is slow.
2. **Main and pilot exciters:** Here, the main exciter is a separately excited DC generator. The field of the main exciter is fed by another DC generator, called the pilot exciter as shown in Fig.2.9(b). Since the main exciter is separately excited, this system has a faster response, as the exciter field control is independent of the exciter output voltage. The pilot exciter may be replaced by a solid state rectifier. The AC supply for this rectifier may be either the main power system or a permanent magnet generator directly connected to the main machine.

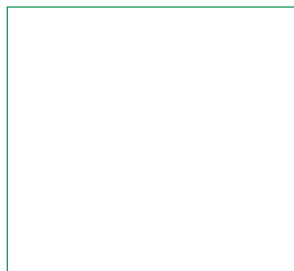


Figure 2.9 DC excitation systems.

#### AC Exciter

An alternator is used to produce AC voltage. The output is rectified and fed to the field winding of the main alternator through slip rings. A fully controlled three-phase bridge rectifier is normally used because of lower voltage stresses on the semiconductors and greater utilization of the transformer capacity. The field winding of the exciter can be fed from a pilot AC exciter with a rectified output, or from a permanent magnet DC generator. The block diagram is shown in Fig. 2.10.

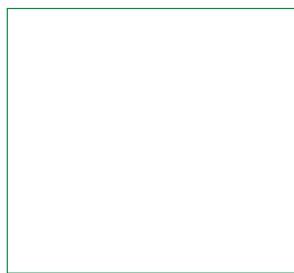


Figure 2.10 AC exciter.

#### Brushless Excitation Systems

Brushless exciters do not require sliprings, commutators, brushes and are practically maintenance free. A commonly used configuration of a brushless exciter is shown in Fig. 2.11.

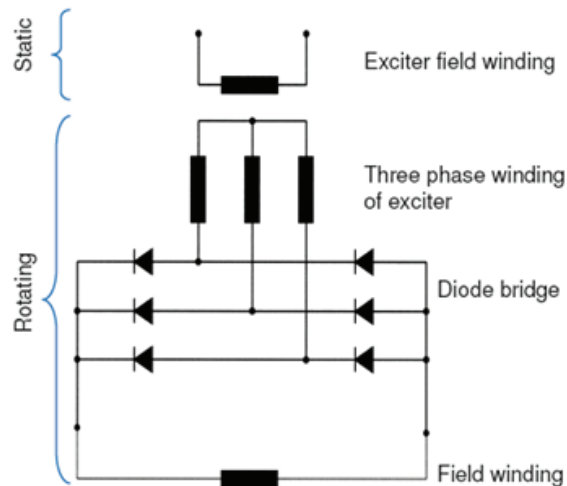


Figure 2.11 Brushless excitation system.

The exciter field winding is supplied by a controlled DC source. This DC could be obtained by using a permanent magnet AC generator and rectifying its output. The exciter field winding induces an AC voltage in the three-phase armature of the exciter. This is rectified by rotating diode wheels and fed directly via leads to the main alternator field winding. This eliminates the brushes and slip rings, which are very expensive in terms of their maintenance cost. Brushless excitation systems are very popular today.

### Static Exciter

These exciters are entirely static. Here, the main generator is self-excited and its output is rectified by means of SCR-rectifier units. The gating of the SCR rectifiers can be controlled using static voltage regulators to generate a control signal. This type of control is very fast, since there is a negligible time delay in shifting the firing angle of the SCRs. The rectified output is fed directly to the field of the main generator through slip rings. The supply of power to the rectifiers is from the main generator or the station auxiliary bus through a step-down transformer. The two common types are potential source-controlled rectifier and compound source rectifiers as shown in Fig. 2.12.

In potential source-controlled rectifiers, the excitation power is supplied through a transformer from the generator terminals - or the station auxiliary bus - and is regulated by a controlled rectifier.

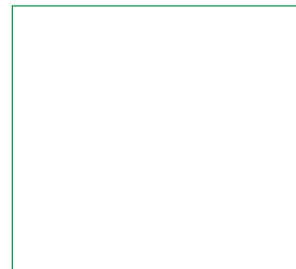


Figure 2.12 Static exciters.

Two independent modes of regulation are provided as follows:

1. The AC regulator automatically maintains the main generator terminal voltage at a desired value corresponding to the AC reference.
2. The DC regulator maintains a constant generator field voltage determined by the DC reference. It is also called the manual control mode and is used when the AC regulator is not operating. This excitation system has a small time constant. The ceiling voltage